Interaction potential among dust grains in a plasma with ion flow

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Linear-response dielectric theory is used to study the interaction potential between dust grains in a flowing plasma, taking into account the finite sizes and the asymmetric charge distributions of the grains. This potential can be divided into two parts: a screened Coulomb potential and a wake potential. The former is a short-ranged repulsive potential, while the later is a long-ranged oscillatory potential which acts only on trailing grains. Both the amplitude and wavelength of the wake potential depend on the Mach number. The grain size and the asymmetric charge distribution may affect the interaction potential in a significant way when the distances between grains are comparable with the grain size, or when the grain size is comparable with the plasma Debye length.

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I. INTRODUCTION

The dusty plasma has attracted much interest in recent years owing to its applications in many areas of research and technology. For example, in the plasma processing of semiconductor manufacturing, it is necessary to control the dust grains to avoid damaging the chips. On the other hand, it has been long known that dust particles exist naturally in space and in the Earth's environment, which is an interesting object in astrophysics and in spacecraft engineering. In particular, the most recent observation of dust lattice formation demonstrated a possibility of using a dusty plasma system as a model system to study condensed matter systems on a ''macroscopic'' level.

Many authors studied extensively various aspects of a dusty plasma, such as charging process of dust in a plasma sheath [1–5], observation of the plasma crystal formation [6–11], wave propagation in dust crystals [12–16], solid-liquid phase transitions in dust crystals [17,18], mechanisms of the dust crystal formation [19–32], etc.

The mechanism of plasma crystal formation is still an open question, although it has been studied extensively, both theoretically and experimentally, over the past several years. In a typical experiment, dust particles are embedded in sheath regions, and thousands of electrons are accumulated on their surfaces. It can then be expected that the resulting Coulomb repulsion among charged dust particles will be very strong. Nevertheless, it has been observed in experiments [30] that ordered Coulomb crystal structures, such as body-centered cubic, face-centered-cubic, and simple hexagonal, are formed by dust particles in the sheath. In particular, for simple hexagonal Coulomb crystals, particles form two-dimensional close-packed structures in the plane parallel to an electrode, and line up along a straight line in the direction perpendicular to the electrode. The ordered crystal structures in the parallel plane may be due to a force balance, i.e., the Coulomb repulsive force between charged grains balancing against the radial confinement force yielded by a copper ring on the electrode. However, it still remains to be clarified what conditions allow the particles to line up regularly on a straight line in the perpendicular direction. To this end, the theory of the wake potential was proposed [19], which is based on collective effects in a plasma with ion flow in the direction perpendicular to the electrode. It was demonstrated [19] that, if the ion-flow speed exceeds the velocity of ion oscillations in the flow, an oscillatory wake potential is formed ''behind'' a static test dust particle, and other dust particles may be trapped in the potential minima. Such an attraction between highly charged dust particles can overcome the Coulomb repulsion, and may explain the mechanism of the dust crystal formation. In recent experiments [30,31], the wake effect on the interaction between dust particles was confirmed by optical manipulations using radiation pressure from laser light.

The wake potential due to a test dust grain is well described by linear dielectric theory, which was used by Vladimirov and co-workers [19–22] in their studies. In particular, they found that, in the near field approximation for the case of supersonic flow ($M>1$), the potential has a cosine dependence on distance, with a wavelength $\lambda_D\sqrt{M^2-1}$, where M is the Mach number and λ_D is the plasma Debye length. Xie *et al.* [24] performed a similar analysis for the case of subsonic flow ($M<1$), which yielded a sine dependence of the potential.

To the best of our knowledge, most theoretical studies of the wake potential to date have considered a dust grain to be a (possibly structured) point charge. This approximation, while neglecting the size effects of the dust grain, provides a reasonable model when the grain size is much smaller than the plasma Debye length, or at sufficiently large distances from the grain. However, a recent observation of the formation of plasma crystals of finite-sized dust grains [33] showed that asymmetric charge distributions on the surface of a dielectric dust grain may affect the structure of the wake potential. In this context, we mention that Lemons *et al.* [25]

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recently considered the size effects of dust grains in two-dimensional geometries by modeling a grain as a charged rod with length L in a planar geometry and a charged disk of radius R in a cylindrical geometry. On the other hand, charge-asymmetry effects were recently studied by Ishihara *et al.* [22], who assumed that a single dust grain is characterized by a point charge Q and a point dipole with a moment p, and deduced that the dipole moment is dominant in forming the wake potential when p becomes of the order of $|Q|\lambda_D$.

In this paper, we study in detail the influence of the dust grain size and the asymmetric charge distribution on the interaction potential among dust grains in a three-dimensional plasma with ion flow, assuming a spherical shape of the grains. The properties of this potential are expected to play an important role in the formation of the dust crystal. After outlining the basic theory of the wake potential in Sec. II, we study two dust grain models with no dipole moments, namely, the point charge (Sec. III) and a uniformly charged sphere (Sec. IV). The effects of the dipole moment are examined in Sec. V, assuming a nonuniform charge distribution on a sphere. Finally, a short summary is given in Sec. VI.

II. BASIC THEORY

We consider a cluster of N dust grains with equal sizes and charge distributions, embedded in a plasma composed of warm electrons and cold ions flowing with the velocity $\mathbf{v}_0$. The presence of dust grains gives rise to a plasma polarization. The scalar potential Φ due to the dust grains is determined by the Poisson equation

$$\nabla^2\Phi(\mathbf{r},t) = -4\pi\left[en_i(\mathbf{r},t) - en_e(\mathbf{r},t) + \sum_{j=1}^{N} \rho_{ext}(\mathbf{r}-\mathbf{r}_j)\right], \tag{1}$$

where $n_i(\mathbf{r},t)$ is the ion density distribution, $n_e(\mathbf{r},t)$ is the electron density distribution, $\rho_{ext}(\mathbf{r}-\mathbf{r}_j)$ is the charge density distribution of a dust grain located at the position $\mathbf{r}_j$, and e is the elemental charge. Within the framework of the hydrodynamics, the ion motion can be described by the continuity equation

$$\frac{\partial n_i(\mathbf{r},t)}{\partial t} + \nabla\cdot[n_i(\mathbf{r},t)\mathbf{u}_i(\mathbf{r},t)] = 0 \tag{2}$$

and the momentum equation

$$\frac{\partial \mathbf{u}_i(\mathbf{r},t)}{\partial t} + \mathbf{u}_i(\mathbf{r},t)\cdot\nabla\mathbf{u}_i(\mathbf{r},t) = -\frac{Z_i e}{m_i}\nabla\Phi(\mathbf{r},t) - \nu\mathbf{u}_i(\mathbf{r},t). \tag{3}$$

Here $\mathbf{u}_i$ is the ion velocity field, m_i is the ion mass, and ν is the friction coefficient due to the neutral gas background. The electrons are characterized simply by the Boltzmann relation $n_e = n_0\exp(e\Phi/T_e)$, or by the equation

$$\nabla n_e(\mathbf{r},t) = \frac{en_e}{T_e}\nabla\Phi(\mathbf{r},t), \tag{4}$$

where T_e is the electron temperature and n_0 is the ion (or electron) density in the plasma, well away from a dust grain.

The system of equations (1)–(4) constitutes a set of self-consistent nonlinear equations, which can only be solved numerically, in general. Since the purpose of the present work is to study wake effects on the interaction potential between dust grains, we can consider the dust charge distribution ρ_{ext} as a weak, first-order perturbation of the plasma fluid. We therefore linearize the above equations by assuming $n_i(\mathbf{r},t) = n_0 + n_{i1}(\mathbf{r},t)$, $\mathbf{u}_i(\mathbf{r},t) = \mathbf{v}_0 + \mathbf{u}_{i1}(\mathbf{r},t)$, $n_e(\mathbf{r},t) = n_0 + n_{e1}(\mathbf{r},t)$, and $\Phi(\mathbf{r},t) = \Phi_1(\mathbf{r},t)$, where $n_{i1}(\mathbf{r},t)$, $\Phi_1(\mathbf{r},t)$, and $\mathbf{u}_{i1}(\mathbf{r},t)$ are perturbed quantities. Using the Fourier transform, one obtains the perturbed potential as

$$\Phi(\mathbf{r}) \equiv \sum_{j=1}^{N} \Phi(\mathbf{r}-\mathbf{r}_j) = \frac{1}{(2\pi)^3}\sum_{j=1}^{N}\int \frac{d\mathbf{k}}{k^2}\frac{4\pi\rho_{ext}(\mathbf{k})}{\varepsilon(k,-\mathbf{k}\cdot\mathbf{v}_0)}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}_j)}, \tag{5}$$

with $\Phi(\mathbf{r}-\mathbf{r}_j)$ being the potential due to the dust grain at the position $\mathbf{r}_j$, $\rho_{ext}(\mathbf{k})$ the Fourier transform of the charge density $\rho_{ext}(\mathbf{r})$, and $\varepsilon(k,-\mathbf{k}\cdot\mathbf{v}_0)$ the dielectric function of the plasma:

$$\varepsilon(k,\omega) = 1 + \frac{1}{(k\lambda_D)^2} - \frac{\omega_{pi}^2}{\omega(\omega+i\nu)}. \tag{6}$$

Here $\omega = -\mathbf{k}\cdot\mathbf{v}_0$, $\lambda_D = (T_e/4\pi n_0 e^2)^{1/2}$ is the electron Debye length, and $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$ is the ion plasma frequency.

The interaction potential between two dust grains placed at $\mathbf{r}_j$ and $\mathbf{r}_l$ is defined by

$$U(\mathbf{r}_{jl}) = \int d\mathbf{r}\,\rho_{ext}(\mathbf{r}-\mathbf{r}_j)\Phi(\mathbf{r}-\mathbf{r}_l) \quad \text{for} \quad j\neq l = 1,2,\ldots,N, \tag{7}$$

where $\mathbf{r}_{jl} = \mathbf{r}_j - \mathbf{r}_l$ is the relative position. On using Eq. (5) in Eq. (7), one obtains

$$U(\mathbf{r}_{jl}) = \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{4\pi|\rho_{ext}(\mathbf{k})|^2}{k^2\varepsilon(k,-\mathbf{k}\cdot\mathbf{v}_0)}e^{i\mathbf{k}\cdot\mathbf{r}_{jl}}. \tag{8}$$

Equation (8) gives a general expression for the interaction potential between two dust grains, depending on both the charge distribution $\rho_{ext}(\mathbf{k})$ and the dielectric function $\varepsilon(k,\omega)$ of the matter. We note that no choice has been made so far for the charge distribution ρ_{ext}. In the following sections, three different models of the charge distribution on a dust grain will be used to calculate the interaction potential.

III. POINT CHARGE

As stated in Sec. I, the point charge model is the simplest model, which gives reasonable results when the plasma Debye length, or the distance between two dust grains, is much larger than the grain radius. In this case, the charge density can be written as

$$\rho_{ext}(\mathbf{r}) = Q\ \delta(\mathbf{r}), \tag{9}$$

where Q is the charge of the dust grain. Thus $\rho_{ext}(\mathbf{k}) = Q$, and Eq. (8) becomes

$$U(\mathbf{r}) = \frac{Q^2}{2\pi^2}\int \frac{d\mathbf{k}}{k^2}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\varepsilon(k,-\mathbf{k}\cdot\mathbf{v}_0)}, \tag{10}$$

where we have dropped the subscript jl in the relative position vector $\mathbf{r}_{jl}$.

Assuming the ion flow velocity $\mathbf{v}_0$ to be oriented along the $-z$ direction, $\mathbf{v}_0 = -v_0\hat{\mathbf{e}}_z$, so that $\omega = -\mathbf{k}\cdot\mathbf{v}_0 = k_z v_0$, we use cylindrical coordinates with $\mathbf{r} = \{\rho,\theta,z\}$ and $\mathbf{k} = \{K,\phi,k_z\}$ to obtain

$$U(\rho,z) = \frac{Q^2}{\pi v_0}\int_0^\infty dK\, K J_0(K\rho)\int_{-\infty}^{\infty} d\omega \frac{e^{i\omega z/v_0}}{k^2\varepsilon(k,\omega)}, \tag{11}$$

where $k^2 = K^2 + (\omega/v_0)^2$, and $J_0(K\rho)$ is the zeroth-order Bessel function. Upon substitution of Eq. (6) into Eq. (11), and using the residue theorem for the ω integration, we obtain

$$U(\rho,z) = U_D(\rho,z) + U_W(\rho,z), \tag{12}$$

where U_D is the screened Coulomb potential,

$$U_D(\rho,z) = U_0\int_0^\infty d\kappa\kappa J_0(\kappa\rho/\lambda_D)\frac{\eta_+}{\eta_+^2+\eta_-^2}e^{-\eta_+|z|/\lambda_D}, \tag{13}$$

and U_W is the wake potential:

$$U_W(\rho,z) = U_0\int_0^\infty d\kappa\kappa J_0(\kappa\rho/\lambda_D)\frac{\eta_-}{\eta_+^2+\eta_-^2} \times \sin(\eta_- z/\lambda_D)\Theta(-z). \tag{14}$$

Here $U_0 = Q^2/\lambda_D$, κ is a dimensionless variable of integration, $\Theta(-z)$ is the unit step function, and

$$\eta_\pm = \frac{1}{\sqrt{2}}\{[(1+\kappa^2-M^{-2})^2+4M^{-2}\kappa^2]^{1/2} \pm(1+\kappa^2-M^{-2})\}^{1/2}, \tag{15}$$

where $M = v_0/v_s$ is the Mach number, $v_s = (T_e/m_i)^{1/2}$ being the ion sound velocity. Obviously, U_W oscillates with z, and exists only behind dust grain, i.e., when $z<0$. In Eqs. (13) and (14), we have assumed the friction coefficient ν, appearing in the dielectric function, to be an infinitesimally small positive quantity. In fact, weak ion-neutral collisions may modify the above results by a factor $\exp[-\nu|z|/(\lambda_D\omega_{pi}M)]$, as shown in Ref. [25]. Although the effects of such collisions on potentials (13) and (14) are interesting, we limit the parameter space in this paper by only considering the situation $\nu \ll \omega_{pi}$. Moreover, it should be mentioned that, due to our use of the hydrodynamic approximation, the effects of Landau damping are also absent in the present theory.

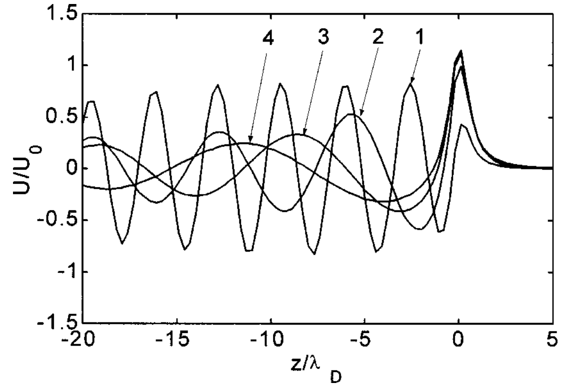

FIG. 1. The interaction potential U/U_0 between two dust grains in the point-charge model as a function of z/λ_D with $\rho = 0.5\lambda_D$. Curves 1, 2, 3, and 4 correspond to $M = 0.5$, 1, 1.5, and 2, respectively.

It follows from Eq. (15) that for a very large Mach number, i.e., $M\to\infty$, η_+ and η_- reduce to $\sqrt{1+\kappa^2}$ and 0, respectively. In this case, U_D reduces to the well-known static Debye potential

$$U_D(r) = \frac{Q^2}{r}e^{-r/\lambda_D}, \tag{16}$$

while the wake potential U_W vanishes, indicating that the asymmetric wake potential will be significant for small Mach numbers. On the other hand, in the case of the far field, i.e., $|z| \gg \lambda_D$, with ρ/λ_D finite, the screened potential U_D quickly approaches zero, while the wake potential continues to oscillate with finite amplitudes. It is believed that exactly this characteristics of the wake potential provides a possibility for dust crystal formation.

Figure 1 shows the variation of the interaction potential U/U_0 with the dimensionless longitudinal distance z/λ_D for $\rho = 0.5\lambda_D$ for several values of the Mach number. One observes that both the wavelength and amplitude of the oscillating potential depend on the Mach number M, such that the former increases with increasing M, while the latter decreases. Note that the wavelength of the potential is about $2\pi M$. To further reveal the characteristics of the wake potential, in Figs. 2(a)–2(c) we plot the dependence of the wake potential on both ρ/λ_D and z/λ_D for three different Mach numbers. One can see that, when the Mach number increases, the spread of the wake potential in the lateral directions becomes narrower or, equivalently, the angle of the Mach cone decreases, which has also been observed by Ishihara and Vladimirov [21].

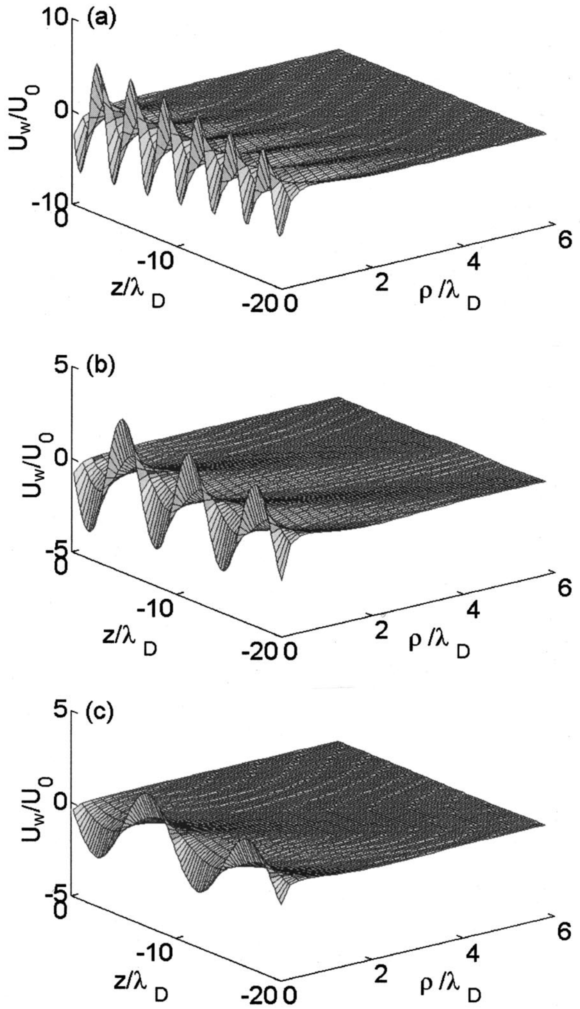

FIG. 2. Profiles of the wake potential U_W/U_0 between two dust grains in the point-charge model for different Mach numbers (a) $M=0.5$, (b) $M=1$, and (c) $M=1.5$.

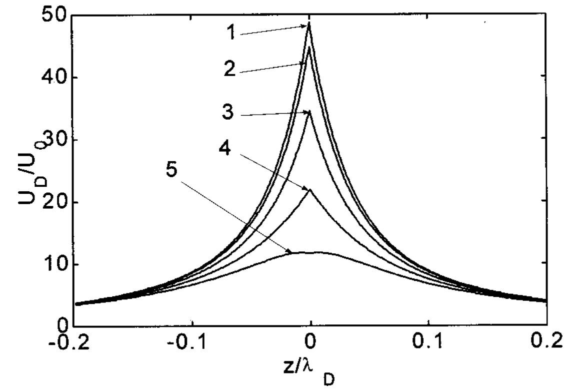

FIG. 3. The screened part U_D/U_0 of the interaction potential between two dust spheres with radius r_c as a function of z/λ_D, with $\rho=2r_c$, and for the Mach number $M=1.5$. Curves 1, 2, 3, and 4 correspond to $r_c=0.$, $0.01\lambda_D$, $0.02\lambda_D$, and $0.03\lambda_D$.

IV. UNIFORM CHARGE DISTRIBUTION

Recently, Lemons *et al.* [25] supposed that the dust grain charge distribution is that of a thin, uniformly charged rod, or disk. In this section, however, we assume that the dust grain is a sphere with a finite radius r_c, having a uniform distribution of charge on its surface, in which case

$$\rho_{ext}(\mathbf{r})=(Q/4\pi r_c^2)\,\delta(r-r_c), \tag{17}$$

and, consequently,

$$\rho_{ext}(\mathbf{k})=Q\,\sin(kr_c)/(kr_c). \tag{18}$$

Following the same procedure as in Sec. III, the interaction potential can also be divided into two parts, $U=U_D+U_W$, with the screened potential given by

$$U_D(\rho,z)=U_0\int_0^\infty d\kappa\kappa J_0(\kappa\rho/\lambda_D)\frac{\eta_+F_1(\kappa)}{\eta_+^2+\eta_-^2}e^{-\eta_+|z|/\lambda_D}, \tag{19}$$

where

$$F_1(\kappa)=\left(\frac{\lambda_D}{r_c}\right)^2\frac{\sin^2[(r_c/\lambda_D)\sqrt{\kappa^2-\eta_+^2}]}{\kappa^2-\eta_+^2}\ \text{for}\ \kappa^2>\eta_+^2$$
$$=\left(\frac{\lambda_D}{r_c}\right)^2\frac{\sinh^2[(r_c/\lambda_D)\sqrt{\eta_+^2-\kappa^2}]}{\eta_+^2-\kappa^2}\ \text{for}\ \kappa^2<\eta_+^2, \tag{20}$$

and the wake potential given by

$$U_W(\rho,z)=U_0\int_0^\infty d\kappa\kappa J_0(\kappa\rho/\lambda_D)\frac{\eta_-F_2(\kappa)}{\eta_+^2+\eta_-^2}\times\sin(\eta_-z/\lambda_D)\Theta(-z), \tag{21}$$

where

$$F_2(\kappa)=\left(\frac{\lambda_D}{r_c}\right)^2\frac{\sin^2[(r_c/\lambda_D)\sqrt{\kappa^2+\eta_-^2}]}{\kappa^2+\eta_-^2}. \tag{22}$$

It is easy to show that each of the functions $F_1(\kappa)$ and $F_2(\kappa)$ approaches unity when $r_c/\lambda_D\to 0$, so that Eqs. (19) and (21) are reduced to Eqs. (13) and (14), respectively, characterizing the point charge model of Sec. III.

In most experiments [28,30–32], the grain radius is about several μm, which is much less than the plasma Debye length, typically of the order of several hundred μm. Nevertheless, in this case it is still worth considering the influence of the grain size on the interaction potential when the distances among two grains are comparable with the grain size. Figure 3 shows the dependence of the screened Coulomb potential U_D on the longitudinal distance z/λ_D with $\rho=2r_c$ and $M=1.5$, for different grain sizes. One can see that the magnitude of the short-range screened potential drops significantly as the grain size increases.

On the other hand, in the space plasma, a dust grain is replaced by a spacecraft with a size which may be comparable to the plasma Debye length. In this case, in Fig. 4 we plot the dependence of the interaction potential U on the longitudinal distance z/λ_D with $\rho=2\lambda_D$ and $M=1.5$ for sev-

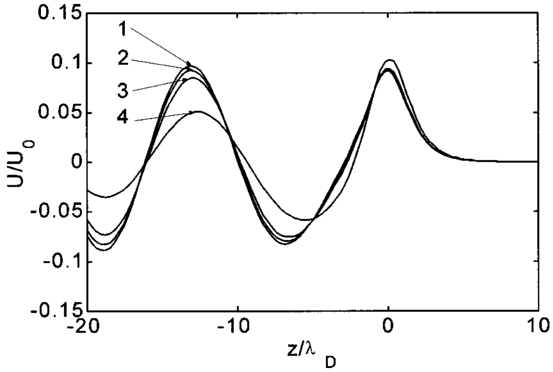

FIG. 4. The interaction potential U/U_0 between two dust spheres with radius r_c as a function of z/λ_D, with $\rho=2\lambda_D$ and for the Mach number $M=1.5$. Curves 1, 2, 3, and 4 correspond to $r_c=0.$, $0.3\lambda_D$, $0.5\lambda_D$, and $1.0\lambda_D$.

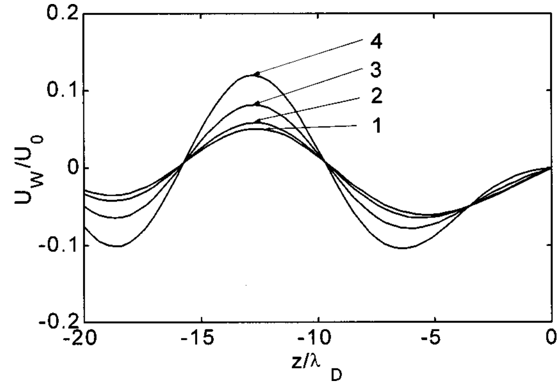

FIG. 5. The wake potential U_W/U_0 between two dust spheres with radius $r_c=\lambda_D$ as a function of z/λ_D, with $\rho=2\lambda_D$ and $M=1.5$, for different dipole moments p. Curves 1, 2, 3, and 4 correspond to $\beta\equiv 3p/(Q\lambda_D)=0$, 2, 4 and 6, respectively.

eral grain sizes. It is clear that the potential is strongly influenced by the grain size in such a way that the oscillations are damped with increasing grain size, and that this influence becomes more prominent in the long-range limit $|z|\gg\lambda_D$.

V. NONUNIFORM CHARGE DISTRIBUTION

It is conceivable that the ion flow must give rise to some asymmetry in the charging processes of dust grains made of dielectric material. In particular, in the case of large-sized dust grains, the effects of ion absorption by grains are important [34], causing greatly enhanced ion fluxes on the grains' upstream faces. Numerical simulations have shown that the ion flow gives rise to an asymmetric charging of grains, with the upstream side of an insulating grain acquiring a more positive charge [35]. Moreover, the sheath electric field may rearrange the charges on the dust grain, also resulting in a nonuniform charge distribution. Although these effects are not described in microscopic detail in the present theory, the resulting asymmetric charging of grains may be treated in a generalized manner by means of a multipolelike expansion. That is, it has been suggested that the grain charge distribution can be characterized by the total charge Q; the dipole moment $\mathbf{p}$, oriented opposite to the ion flow velocity $\mathbf{v}_0$, such that $\mathbf{p}=p\,\hat{\mathbf{e}}_z$, and possibly by the higher-order multipoles as well [22].

Here we again consider a spherical dust grain of radius r_c with the surface charge density which can be expanded in terms of the direction cosine $\cos\vartheta_{\mathbf{r}}=\hat{\mathbf{e}}_z\cdot\mathbf{r}/r$, such that

$$\rho_{ext}(\mathbf{r})=\frac{Q}{4\pi r_c^2}\,\delta(r-r_c)(1+\alpha\cos\vartheta_{\mathbf{r}}+\cdots), \tag{23}$$

where $\alpha=3p/(r_cQ)$. It has been shown [35] that the dipole moment p can reach quite large values with increasing Mach number, so that the parameter α may easily exceed the value of, say, 10. Since the dipole term will have the longest range of all the multipoles in the induced potential, we retain only the dipole term and obtain the Fourier transform of the grain charge density as follows:

$$\rho_{ext}(\mathbf{k})=Q\left\{\frac{\sin(kr_c)}{kr_c}+i\frac{\alpha}{kr_c}\cos\vartheta_{\mathbf{k}}\left[\cos(kr_c)-\frac{\sin(kr_c)}{kr_c}\right]\right\}, \tag{24}$$

where $\cos\vartheta_{\mathbf{k}}=\hat{\mathbf{e}}_z\cdot\mathbf{k}/k$.

Again following the procedure outlined in Sec. III, the interaction potential among two grains can be written as a sum of the screened Coulomb potential $U_D(\rho,z)$ and the wake potential $U_W(\rho,z)$, which are given by Eqs. (19) and (21), respectively, with the functions $F_1(\kappa)$ and $F_2(\kappa)$ now being defined as follows:

$$F_1(\kappa)=\frac{\sin^2\Delta(\kappa)}{\Delta^2(\kappa)}-\frac{\beta^2\eta_+^2}{\Delta^4(\kappa)}\left[\cos\Delta(\kappa)-\frac{\sin\Delta(\kappa)}{\Delta(\kappa)}\right]^2 \quad \text{for } \kappa^2>\eta_+^2,$$

$$=\frac{\sinh^2\Gamma(\kappa)}{\Gamma^2(\kappa)}-\frac{\beta^2\eta_+^2}{\Gamma^4(\kappa)}\left[\cosh\Gamma(\kappa)-\frac{\sinh\Gamma(\kappa)}{\Gamma(\kappa)}\right]^2 \quad \text{for } \kappa^2<\eta_+^2, \tag{25}$$

and

$$F_2(\kappa)=\frac{\sin^2\Pi(\kappa)}{\Pi^2(\kappa)}+\frac{\beta^2\eta_-^2}{\Pi^4(\kappa)}\left[\cos\Pi(\kappa)-\frac{\sin\Pi(\kappa)}{\Pi(\kappa)}\right]^2, \tag{26}$$

where $\Delta(\kappa)=(r_c/\lambda_D)\sqrt{\kappa^2-\eta_+^2}$, $\Gamma(\kappa)=(r_c/\lambda_D)\sqrt{\eta_+^2-\kappa^2}$, $\Pi(\kappa)=(r_c/\lambda_D)\sqrt{\kappa^2+\eta_-^2}$, and $\beta\equiv\alpha r_c/\lambda_D=3p/(Q\lambda_D)$. In Fig. 5 we plot the dependence of the wake potential U_W/U_0 on the longitudinal distance z/λ_D with $r_c=\lambda_D$, $\rho=2\lambda_D$ and $M=1.5$ for several values of the parameter β. One can see that the effects of the dipole moment on the potential are quite strong, so that the amplitudes of oscillations increase with increasing β, and this influence seems to be more prominent at large distances.

It is interesting to further investigate the role played by grain size in the effects of the dipole moment on the poten-

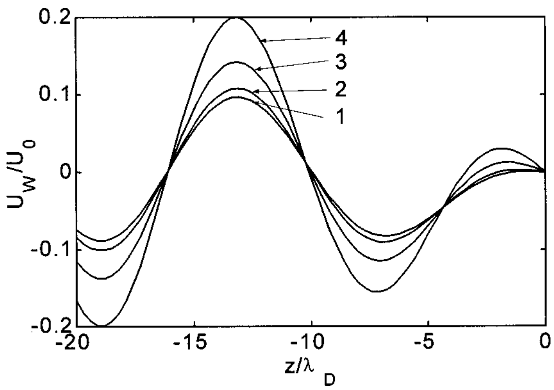

FIG. 6. The wake potential U_W/U_0 between two point-dust grains with radius $r_c=0$, as a function of z/λ_D, with $\rho=2\lambda_D$ and $M=1.5$, for different dipole moments p. Curves 1, 2, 3, and 4 correspond to $\beta\equiv 3p/(Q\lambda_D)=0$, 2, 4, and 6, respectively.

tial. To this end, we recover a point charge with a point-dipole model of Ref. [22] by considering the situation $r_c \ll \lambda_D$, in which case

$$\lim_{r_c\to 0} \rho_{ext}(\mathbf{k})=Q-i\,\mathbf{p}\cdot\mathbf{k}. \tag{27}$$

In this limit, the functions in Eqs. (25) and (26) become, respectively, $F_1(\kappa)=1-(\beta\eta_+/3)^2$ and $F_2(\kappa)=1+(\beta\eta_-/3)^2$. Using these expressions in Eqs. (19) and (21) gives the potential components in the point charge with a point-dipole model [22]. In Fig. 6 we plot the wake potential U_W/U_0 in this model as a function of the longitudinal distance z/λ_D, with all the parameters being identical to those in Fig. 5, except that now $r_c=0$. Interestingly, the shapes of all the curves resemble those in Fig. 5, except that the amplitudes of the oscillations are much larger in the $r_c=0$ case than for a finite-size grain, for all values of the dipole-moment parameter β displayed. This conclusion is in accord with the effects of the increasing grain size displayed in Fig. 4 in the zero-dipole case.

VI. SUMMARY

We have studied the characteristics of the interaction potential between two dust grains embedded in a flowing plasma. A general expression for the potential has been derived within the linear hydrodynamics model, taking into account the charge distributions of the grains. The potential is divided into two parts: a screened Coulomb potential and a wake potential. The former is a repulsive potential which drops very quickly with increasing distance between the grains, while the latter is an oscillatory, long-range potential which affects only those grains which are trailing downstream relative to the ion flow. Numerical results show that the Mach number M is a key parameter in describing the interaction among the grains, in such a manner that the wavelength of the oscillating potential increases, while its amplitude decreases, with increasing M.

In order to go beyond the point-charge model, the grain size effects on the potential have been considered by assuming the grain to be a sphere with a uniform charge distribution on its surface. It has been shown that these effects are significant when the grain size is comparable with the plasma Debye length and/or with the distance among the grains. Moreover, the effects of the asymmetric charge distribution on the interaction potential have been investigated by means of a model of a nonuniformly charged sphere, taking into account the terms up to and including the dipole term. It has been shown that the dipole effects on the potential are quite strong when the dipole moment p exceeds the value $|Q|\lambda_D$. The main conclusion is that the dipole effects are fairly long ranged, and generally increase the amplitudes of the oscillating potential, while the increasing size of the grain essentially reduces those amplitudes. Thus further studies of dust crystal formation in plasmas need to carefully examine the roles played by the dust grain size (and possibly the shape as well) and, in particular, the nonuniformity of the charge accumulated on the grain surfaces.

In future work, we will extend the present model to study the influences of the interaction potential on the motion of the dust grains. In addition, we plan to study a fully self-consistent problem of dust charging in the presence of nearby dust grains in a flowing plasma.

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